The spreading speeds of disturbance in a nonlocal Fisher equation

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Abstract

We consider the nonlocal analogue of the Fisher equation

$$u_t = \mu * u - u + u(1-u),$$

where μ is a probability distribution. We show that if an initial disturbance extends widely, then the disturbance spreads. Further, we give a formula of the spreading speeds.

Keywords: convolution model, integro-differential equation, discrete monostable equation, nonlocal monostable equation, nonlocal Fisher-KPP equation

1. Introduction

In 1930, Fisher [8] introduced the reaction-diffusion equation $u_t = u_{xx} + u(1-u)$ as a model for the spatial spread of an advantageous form of a single gene in a population. He [9] found that there is a constant c_* such that the equation has a traveling wave solution with speed c when $c \geq c_*$ while it has no such solution when $c < c_*$. Kolmogorov, Petrovsky and Piskunov [16] obtained the same conclusion for a monostable equation $u_t = u_{xx} + f(u)$ with a more general nonlinearity f, and investigated long-time behavior of this model. Since these pioneering works, there have been extensive studies on traveling waves and long-time behavior for monostable evolution systems.

In this paper, we consider the following nonlocal analogue of the Fisher equation:

$$u_t = \mu * u - u + u(1-u). \tag{1.1}$$

Here, μ is a Borel-measure on $\mathbb{R}$ with $\mu(\mathbb{R}) = 1$ and the convolution is defined by

$$(\mu * u)(x) := \int_{y\in\mathbb{R}} u(x-y)d\mu(y)$$

for a bounded and continuous function u on $\mathbb{R}$. We would show that if an initial disturbance extends widely, then the disturbance spreads with certain speeds $c_\pm$, which are formulated in Theorem 1. The main result of this paper is the following:

Theorem 1. *Suppose $\mu((0,+\infty)) \neq 0$ and there is a positive constant λ satisfying $\int_{y\in\mathbb{R}} e^{\lambda|y|}d\mu(y) < +\infty$. Let two nonnegative constants c_- and c_+ be defined by*

$$c_- := \inf_{\lambda>0} \frac{1}{\lambda} \int_{y\in\mathbb{R}} e^{-\lambda y} d\mu(y)$$

and

$$c_+ := \inf_{\lambda>0} \frac{1}{\lambda} \int_{y\in\mathbb{R}} e^{+\lambda y} d\mu(y).$$

Then, $c_+ > 0$ *and the following two hold:*

(i) *Let* τ *be a positive constant, and* I' *an open interval which contains* $[-c_-, +c_+]$. *Suppose that a continuous function* u_0 *on* $\mathbb{R}$ *has a compact support and* $0 \leq u_0(x) < 1$ *holds for all* $x \in \mathbb{R}$. *Then, the solution* $u(t, x)$ *to* (1.1) *with* $u(0, x) \equiv u_0(x)$ *satisfies*

$$\lim_{n\to\infty} \sup_{x\in\mathbb{R}\setminus I'} u(n\tau, n\tau x) = 0.$$

(ii) *Let* τ *be a positive constant, and* I'' *a closed interval which is contained in* $(-c_-, +c_+)$. *For any* $\sigma > 0$, *there exists* $r > 0$ *satisfying the following. Suppose that* u_0 *is a continuous function on* $\mathbb{R}$, $0 \leq u_0(x) \leq 1$ *holds for all* $x \in \mathbb{R}$ *and* $\sigma \leq u_0(x)$ *holds for all* $x \in [-r, +r]$. *Then, the solution* $u(t, x)$ *to* (1.1) *with* $u(0, x) \equiv u_0(x)$ *satisfies*

$$\lim_{n\to\infty} \inf_{x\in I''} u(n\tau, n\tau x) = 1.$$

In order to prove Theorem 1, we employ theorems by Weinberger [25]. We do not assume that the probability measure μ is absolutely continuous with respect to the Lebesgue measure. For example, not only the integro-differential equation

$$\frac{\partial u}{\partial t}(t, x) = \int_0^1 u(t, x - y)dy - u(t, x)^2$$

but also the discrete Fisher equation

$$\frac{\partial u}{\partial t}(t, x) = u(t, x - 1) - u(t, x)^2$$

satisfies the assumption of Theorem 1.

See, e.g., [1, 3, 5, 6, 7, 10, 11, 12, 13, 14, 15, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28] on traveling waves and long-time behavior in various monostable evolution systems, [2, 4] nonlocal bistable equations and [20] the Euler equation.

2. Proof of Theorem 1

Let $BC(\mathbb{R})$ denote the Banach space of bounded and continuous functions on $\mathbb{R}$ with the supremum norm.

We first state that the time τ map of the semiflow generated by some nonlocal equation is continuous with respect to the compact-open topology.

Lemma 2. *Let* τ *be a positive constant,* $\hat{\mu}$ *a Borel-measure on* $\mathbb{R}$ *and* g *a Lipschitz continuous function on* $\mathbb{R}$. *Suppose there exists a positive constant* $\hat{\lambda}$ *satisfying* $\int_{y\in\mathbb{R}} e^{\hat{\lambda}|y|} d\hat{\mu}(y) < +\infty$. *Let* $\{v_n\}_{n=0}^{\infty} \subset C^1([0, \tau], BC(\mathbb{R}))$ *be a sequence of solutions to the equation*

$$v_t = \hat{\mu} * v + g(v).$$

Suppose $\sup_{n\in\mathbb{N},x\in\mathbb{R}} |v_n(0,x)| < +\infty$. *Then,* $v_n(0,x) \to v_0(0,x)$ *as* $n \to \infty$ *uniformly in* x *on every bounded interval implies* $v_n(\tau,x) \to v_0(\tau,x)$ *as* $n \to \infty$ *uniformly in* x *on every bounded interval.*

Proof. See, e.g., Proposition 19 in [28]. □

The following is the main technical result, and it is proved in Section 3.

Lemma 3. *Let* τ *be a positive constant and* $\hat{\mu}$ *a Borel-measure on* $\mathbb{R}$. *Suppose there exists a positive constant* $\hat{\lambda}$ *satisfying* $\int_{y\in\mathbb{R}} e^{\hat{\lambda}|y|}d\hat{\mu}(y) < +\infty$. *Let* $\hat{P} : BC(\mathbb{R}) \to BC(\mathbb{R})$ *be the time* τ *map of the flow on* $BC(\mathbb{R})$ *generated by the linear equation*

$$v_t = \hat{\mu} * v. \tag{2.1}$$

Then, there exists a Borel-measure $\hat{\nu}$ *on* $\mathbb{R}$ *with* $\hat{\nu}(\mathbb{R}) < +\infty$ *such that*

$$\hat{P}[v] = \hat{\nu} * v$$

holds for all $v \in BC(\mathbb{R})$. *Further, the equality*

$$\log \int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) = \left(\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}(y)\right)\tau \tag{2.2}$$

holds for all $\lambda \in \mathbb{R}$.

Let $\mathcal{B}$ denote the set of continuous functions u on $\mathbb{R}$ with $0 \le u \le 1$.

We could obtain the following by the comparison theorem.

Lemma 4. *Let* τ *be a positive constant and* μ *a Borel-measure on* $\mathbb{R}$ *with* $\mu(\mathbb{R}) = 1$. *Let* $P : BC(\mathbb{R}) \to BC(\mathbb{R})$ *be the time* τ *map of the flow on* $BC(\mathbb{R})$ *generated by the linear equation*

$$v_t = \mu * v \tag{2.3}$$

and $Q : \mathcal{B} \to \mathcal{B}$ *the time* τ *map of the semiflow on* $\mathcal{B}$ *generated by the Fisher equation*

$$v_t = \mu * v - v^2. \tag{2.4}$$

Then, the following two hold:

(i) *The inequality*

$$Q[u] \le P[u]$$

holds for all $u \in \mathcal{B}$.

(ii) *For any* $\delta \in (0,1)$, *there exists* $\varepsilon \in (0,1)$ *such that for any* $u \in \mathcal{B}$ *with* $0 \le u \le \varepsilon$, *the inequality*

$$(1-\delta)P[u] \le Q[u]$$

holds.

Proof. By the comparison theorem between (2.3) and (2.4), we have $Q[u] \le P[u]$ for all $u \in \mathcal{B}$.

We take a positive constant ε as

$$\varepsilon := \min\left\{\left(-\frac{1}{\tau}\log(1-\delta)\right)e^{-\tau}, \frac{1}{2}\right\}.$$

Let a function $u \in \mathcal{B}$ satisfy $0 \le u \le \varepsilon$. Then, we take the solution $v(t,x)$ with $v(0,x) \equiv u(x)$ to the linear equation

$$v_t = \mu * v + \left(\frac{1}{\tau}\log(1-\delta)\right)v. \tag{2.5}$$

So, we have

$$(1-\delta)(P[u])(x) \equiv v(\tau, x).$$

Because

$$0 \le v(t,x) \le \varepsilon e^t \le \left(-\frac{1}{\tau}\log(1-\delta)\right)$$

holds for all $t \in [0,\tau]$, we see

$$\left(\frac{1}{\tau}\log(1-\delta)\right)v(t,x) \le -v(t,x)^2$$

for all $t \in [0,\tau]$. Hence, by the comparison theorem between (2.4) and (2.5),

$$(1-\delta)(P[u])(x) \equiv v(\tau,x) \le (Q[u])(x)$$

holds. □

In virtue of Lemmas 2, 3 and 4, we could apply Theorems 6.1, 6.2 and Corollary in Section 6 of [25] to prove Theorem 1.

Proof of Theorem 1. Let $P : BC(\mathbb{R}) \to BC(\mathbb{R})$ be the time τ map of the flow on $BC(\mathbb{R})$ generated by the linear equation

$$u_t = \mu * u$$

and $Q : \mathcal{B} \to \mathcal{B}$ the time τ map of the semiflow on $\mathcal{B}$ generated by the Fisher equation

$$u_t = \mu * u - u^2.$$

Then, from Lemma 3, there exists a Borel-measure ν on $\mathbb{R}$ with $\nu(\mathbb{R}) < +\infty$ such that

$$P[u] = \nu * u \tag{2.6}$$

holds for all $u \in BC(\mathbb{R})$. Further, the equality

$$\log \int_{y\in\mathbb{R}} e^{\lambda y} d\nu(y) = \left(\int_{y\in\mathbb{R}} e^{\lambda y} d\mu(y)\right)\tau$$

holds for all $\lambda \in \mathbb{R}$. From this equality, we have

$$c_-^* := c_-\tau = \inf_{\lambda>0} \frac{1}{\lambda}\log\int_{y\in\mathbb{R}} e^{-\lambda y} d\nu(y)$$

and

$$c_+^* := c_+\tau = \inf_{\lambda>0} \frac{1}{\lambda}\log\int_{y\in\mathbb{R}} e^{+\lambda y} d\nu(y).$$

By Lemma 4 and (2.6), the inequality

$$Q[u] \le P[u] = \nu * u$$

holds for all $u \in \mathcal{B}$. For any $\delta \in (0,1)$, there exists $\varepsilon \in (0,1)$ such that for any $u \in \mathcal{B}$ with $0 \le u \le \varepsilon$, the inequality

$$(1-\delta)\nu * u = (1-\delta)P[u] \le Q[u]$$

holds. From Lemma 2, with $\pi_0 := 0$, $\pi_1 := 1$ and $\mathcal{H} := \mathbb{R}$, we also see that Q satisfies the hypotheses (3.1) in [25]. Therefore, with $N := 1$ and $S^{N-1} := \{\pm 1\}$, we obtain the conclusion of Theorem 1 by applying Theorems 6.1, 6.2 and Corollary in Section 6 of [25], because of $[-c_-^*, +c_+^*] \subset \tau I'$ and $\tau I'' \subset (-c_-^*, +c_+^*)$. □

3. Proof of Lemma 3

[Step 1] In this step, we show the following: *There exists a Borel-measure $\hat{\nu}$ on $\mathbb{R}$ with $\hat{\nu}(\mathbb{R}) < +\infty$ such that*

$$\hat{P}[v] = \hat{\nu} * v$$

holds for all $v \in BC(\mathbb{R})$.

Put a functional $P : BC(\mathbb{R}) \to \mathbb{R}$ as

$$P[v] := (\hat{P}[v])(0).$$

Then, the functional P is linear, bounded and positive. Hence, there exists a Borel-measure ν on $\mathbb{R}$ with $\nu(\mathbb{R}) < +\infty$ such that if a function $v \in BC(\mathbb{R})$ satisfies $\lim_{|x|\to\infty} v(x) = 0$, then

$$P[v] = \int_{y\in\mathbb{R}} v(y)d\nu(y) \tag{3.1}$$

holds.

Let $v \in BC(\mathbb{R})$. Then, there exists a sequence $\{v_n\}_{n=1}^{\infty} \subset BC(\mathbb{R})$ with $\sup_{n\in\mathbb{N},x\in\mathbb{R}} |v_n(x)| < +\infty$ and $\lim_{|x|\to\infty} v_n(x) = 0$ for all $n \in \mathbb{N}$ such that $v_n \to v$ as $n \to \infty$ uniformly on every bounded interval. From Lemma 2, (3.1) and $\nu(\mathbb{R}) < +\infty$, we have

$$P[v] = \lim_{n\to\infty} P[v_n] = \lim_{n\to\infty} \int_{y\in\mathbb{R}} v_n(y)d\nu(y) = \int_{y\in\mathbb{R}} v(y)d\nu(y).$$

We take a Borel-measure $\hat{\nu}$ on $\mathbb{R}$ with $\hat{\nu}(\mathbb{R}) < +\infty$ such that

$$\hat{\nu}((-\infty, y)) = \nu((-y, +\infty))$$

holds for all $y \in \mathbb{R}$. Then, for any $v \in BC(\mathbb{R})$, we have

$$(\hat{P}[v])(x) \equiv P[v(\cdot + x)] \equiv \int_{y\in\mathbb{R}} v(y+x)d\nu(y) \equiv (\hat{\nu} * v)(x).$$

[Step 2] We show the following: *The equality (2.2) holds when $\lambda = 0$.*

Because

$$e^{\left(\int_{y\in\mathbb{R}} 1d\hat{\mu}(y)\right)t}$$

is a solution to (2.1), by Step 1, we see

$$\int_{y\in\mathbb{R}} 1d\hat{\nu}(y) = (\hat{\nu} * 1)(0) = (\hat{P}[1])(0) = e^{\left(\int_{y\in\mathbb{R}} 1d\hat{\mu}(y)\right)\tau}.$$

[Step 3] We show the following: *The equality*

$$\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) = \lim_{n\to\infty} (\hat{P}[\min\{e^{-\lambda x}, n\}])(0)$$

holds for all $\lambda \in \mathbb{R}$.

In virtue of Step 1, we have

$$\begin{aligned}\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) &= \lim_{n\to\infty} \int_{y\in\mathbb{R}} \min\{e^{\lambda y}, n\} d\hat{\nu}(y) \\ &= \lim_{n\to\infty} (\hat{\nu} * \min\{e^{-\lambda x}, n\})(0) = \lim_{n\to\infty} (\hat{P}[\min\{e^{-\lambda x}, n\}])(0).\end{aligned}$$

[Step 4] We show the following: *If a constant* $\lambda \in \mathbb{R} \setminus \{0\}$ *satisfies* $\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}(y) < +\infty$*, then the equality (2.2) holds.*

Let X be the set of continuous functions u on $\mathbb{R}$ with $\sup_{x\in\mathbb{R}} \frac{|u(x)|}{1+e^{-\lambda x}} < +\infty$. Then, X is a Banach space with the norm $\|u\|_X := \sup_{x\in\mathbb{R}} \frac{|u(x)|}{1+e^{-\lambda x}}$. We have

$$\begin{aligned}\|\hat{\mu} * u\|_X &\le \sup_{x\in\mathbb{R}} \frac{\int_{y\in\mathbb{R}} |u(x-y)| d\hat{\mu}(y)}{1+e^{-\lambda x}} \\ &\le \sup_{x\in\mathbb{R}} \int_{y\in\mathbb{R}} \frac{|u(x-y)|}{1+e^{-\lambda(x-y)}} (1+e^{\lambda y}) d\hat{\mu}(y) \\ &\le \left(\int_{y\in\mathbb{R}} (1+e^{\lambda y}) d\hat{\mu}(y)\right) \|u\|_X.\end{aligned}$$

Let $\hat{P}_X : X \to X$ be the time τ map of the flow on X generated by the linear equation (2.1).

Suppose $\lambda > 0$. Let $\bar{\lambda} \in (0, \lambda)$. Then, we see

$$\begin{aligned}\lim_{n\to\infty} \|\min\{e^{-\bar{\lambda} x}, n\} - e^{-\bar{\lambda} x}\|_X &\le \lim_{n\to\infty} \sup_{x\in(-\infty, -\frac{1}{\bar{\lambda}} \log n)} \frac{e^{-\bar{\lambda} x}}{1+e^{-\lambda x}} \\ &\le \lim_{n\to\infty} \sup_{x\in(-\infty, -\frac{1}{\bar{\lambda}} \log n)} e^{(\lambda-\bar{\lambda})x} = 0.\end{aligned}$$

Hence, by Step 3,

$$\begin{aligned}\int_{y\in\mathbb{R}} e^{\bar{\lambda} y} d\hat{\nu}(y) &= \lim_{n\to\infty} (\hat{P}[\min\{e^{-\bar{\lambda} x}, n\}])(0) \\ &= \lim_{n\to\infty} (\hat{P}_X[\min\{e^{-\bar{\lambda} x}, n\}])(0) = (\hat{P}_X[e^{-\bar{\lambda} x}])(0) = e^{\left(\int_{y\in\mathbb{R}} e^{\bar{\lambda} y} d\hat{\mu}(y)\right)\tau},\end{aligned}$$

because

$$e^{\left(\int_{y\in\mathbb{R}} e^{\bar{\lambda} y} d\hat{\mu}(y)\right)t - \bar{\lambda} x}$$

is a solution to (2.1). So, we have

$$\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) = \lim_{\bar{\lambda}\uparrow\lambda} \int_{y\in\mathbb{R}} e^{\bar{\lambda} y} d\hat{\nu}(y) = \lim_{\bar{\lambda}\uparrow\lambda} e^{\left(\int_{y\in\mathbb{R}} e^{\bar{\lambda} y} d\hat{\mu}(y)\right)\tau} = e^{\left(\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}(y)\right)\tau}.$$

When $\lambda < 0$, we could also prove it almost similarly as $\lambda > 0$.

[Step 5] It is sufficient to conclude the proof of Lemma 3, if we show that $\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}(y) = +\infty$ implies $\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) = +\infty$.

For each $n \in \mathbb{N}$, let $\hat{P}_n : BC(\mathbb{R}) \to BC(\mathbb{R})$ be the time τ map of the flow on $BC(\mathbb{R})$ generated by the linear equation

$$v_t = \hat{\mu}_n * v, \tag{3.2}$$

where $\hat{\mu}_n$ is the Borel-measure on $\mathbb{R}$ such that

$$\hat{\mu}_n((-\infty, y)) = \hat{\mu}((-\infty, y) \cap (-n, +n))$$

holds for all $y \in \mathbb{R}$. Then, in virtue of Step 1, there exists a Borel-measure $\hat{\nu}_n$ on $\mathbb{R}$ with $\hat{\nu}_n(\mathbb{R}) < +\infty$ such that

$$\hat{P}_n[v] = \hat{\nu}_n * v$$

holds for all $v \in BC(\mathbb{R})$. Further, by Steps 2 and 4, we also have

$$\log \int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}_n(y) = \left(\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}_n(y)\right)\tau = \left(\int_{y\in(-n,+n)} e^{\lambda y} d\hat{\mu}(y)\right)\tau.$$

Therefore, because a nonnegative solution to (3.2) is a sub-solution to (2.1), by Step 3, we obtain the inequality

$$\begin{aligned}\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) &= \lim_{m\to\infty} (\hat{P}[\min\{e^{-\lambda x}, m\}])(0) \\ &\geq \lim_{m\to\infty} (\hat{P}_n[\min\{e^{-\lambda x}, m\}])(0) = \lim_{m\to\infty} \int_{y\in\mathbb{R}} \min\{e^{\lambda y}, m\} d\hat{\nu}_n(y) \\ &= \int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}_n(y) = e^{\left(\int_{y\in(-n,+n)} e^{\lambda y} d\hat{\mu}(y)\right)\tau}\end{aligned}$$

for all $n \in \mathbb{N}$. Hence, $\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\mu}(y) = +\infty$ implies $\int_{y\in\mathbb{R}} e^{\lambda y} d\hat{\nu}(y) = +\infty$. □

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References

[1] C. Atkinson and G. E. H. Reuter, Deterministic epidemic waves, *Math. Proc. Cambridge Philos. Soc.*, 80 (1976), 315–330.

[2] P. W. Bates, P. C. Fife, X. Ren and X. Wang, Traveling waves in a convolution model for phase transitions, *Arch. Rational Mech. Anal.*, 138 (1997), 105–136.

[3] J. Carr and A. Chmaj, Uniqueness of travelling waves for nonlocal monostable equations, *Proc. Amer. Math. Soc.*, 132 (2004), 2433–2439.

[4] X. Chen, Existence, uniqueness, and asymptotic stability of traveling waves in nonlocal evolution equations, *Adv. Differential Equations*, 2 (1997), 125–160.

[5] X. Chen and J.-S. Guo, Uniqueness and existence of traveling waves for discrete quasilinear monostable dynamics, *Math. Ann.*, 326 (2003), 123–146.

[6] J. Coville, J. Dávila and S. Martínez, Nonlocal anisotropic dispersal with monostable nonlinearity, *J. Differential Equations*, 244 (2008), 3080–3118.

[7] J. Coville and L. Dupaigne, On a non-local equation arising in population dynamics, *Proc. Roy. Soc. Edinburgh A*, 137 (2007), 727–755.

[8] R. A. Fisher, *The Genetical Theory of Natural Selection*, Clarendon Press, Oxford, 1930.

[9] R. A. Fisher, The wave of advance of advantageous genes, *Ann. Eugenics*, 7 (1937), 335–369.

[10] B. H. Gilding and R. Kersner, *Travelling Waves in Nonlinear Diffusion-Convection Reaction*, Birkhäuser, Basel, 2004.

[11] J.-S. Guo and F. Hamel, Front propagation for discrete periodic monostable equations, *Math. Ann.*, 335 (2006), 489–525.

[12] J.-S. Guo and Y. Morita, Entire solutions of reaction-diffusion equations and an application to discrete diffusive equations, *Discrete Contin. Dynam. Systems*, 12 (2005), 193–212.

[13] F. Hamel and N. Nadirashvili, Entire solutions of the KPP equation, *Comm. Pure Appl. Math.*, 52 (1999), 1255–1276.

[14] Y. Hosono, The minimal speed for a diffusive Lotka-Volterra model, *Bull. Math. Biol.*, 60 (1998), 435–448.

[15] Y. Kan-on, Fisher wave fronts for the Lotka-Volterra competition model with diffusion, *Nonlinear Anal.*, 28 (1997), 145–164.

[16] A. N. Kolmogorov, I. G. Petrovsky and N. S. Piskunov, Étude de l'équation de la difusion avec croissance de la quantité de matière et son application à un problème biologique, *Bull. Univ. Moskov. Ser. Internat. A*, 1 (1937), 1–25.

[17] B. Li, H. F. Weinberger and M. A. Lewis, Spreading speeds as slowest wave speeds for cooperative systems, *Math. Biosci.*, 196 (2005), 82–98.

[18] X. Liang, Y. Yi and X.-Q. Zhao, Spreading speeds and traveling waves for periodic evolution systems, *J. Differential Equations*, 231 (2006), 57–77.

[19] X. Liang and X.-Q. Zhao, Asymptotic speeds of spread and traveling waves for monotone semiflows with applications, *Comm. Pure Appl. Math.*, 60 (2007), 1–40.

[20] H. Okamoto and M. Shoji, *The Mathematical Theory of Permanent Progressive Water-Waves*, World Scientific Publishing Co., River Edge, 2001.

[21] K. Schumacher, Travelling-front solutions for integro-differential equations. I, *J. Reine Angew. Math.*, 316 (1980), 54–70.

[22] K. Schumacher, Travelling-front solutions for integrodifferential equations II, *Biological Growth and Spread*, pp. 296–309, Springer, Berlin-New York, 1980.

[23] K. Uchiyama, The behavior of solutions of some nonlinear diffusion equations for large time, *J. Math. Kyoto Univ.*, 18 (1978), 453–508.

[24] A. I. Volpert, V. A. Volpert and V. A. Volpert, *Traveling wave solutions of parabolic systems*, American Mathematical Society, Providence, 1994.

[25] H. F. Weinberger, Long-time behavior of a class of biological models, *SIAM J. Math. Anal.*, 13 (1982), 353–396.

[26] H. F. Weinberger, On spreading speeds and traveling waves for growth and migration models in a periodic habitat, *J. Math. Biol.*, 45 (2002), 511–548.

[27] H. F. Weinberger, M. A. Lewis and B. Li, Analysis of linear determinacy for spread in cooperative models, *J. Math. Biol.*, 45 (2002), 183–218.

[28] H. Yagisita, Existence of traveling waves for a nonlocal monostable equation: an abstract approach, preprint (http://arxiv.org/pdf/0807.3612).

非局所フィッシャー方程式における擾乱の伝播速度

柳　下　浩　紀

要　旨

本論文では，非局所フィッシャー方程式

$$u_t = \mu * u - u + u(1 - u)$$

を考察する．ここで，μ は確率分布である．初期の擾乱が広範囲に渡れば擾乱が伝播していくことを示し，さらに伝播速度の公式を与える．

キーワード： 合成積モデル，微分積分方程式，離散単安定方程式，非局所単安定方程式，非局所フィッシャー・KPP 方程式