Why Exchange Rate is Indeterminate in Open Macro Economic Models?
—An Approach from General Equilibrium View Point—

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SUMMARY

Every economic models of exchange rate have failed to explain even the medium term exchange rate movements. They still have failed to outperform the naïve random walk model. This paper tries to clarify a part of the problem chiefly from accounting point of view. When the complete stock-flow accounting is incorporated, the two-country open economy Portfolio Balance Model has just two independent equations for asset market clearing. It can determine home and foreign interest rates but not the exchange rate. If asset market equilibria vary smoothly over time, the balance of payments equation in the Mundell–Fleming model is not independent and cannot set the exchange rate either, and "fundamentals-based" econometric models of the exchange rate would almost certainly fail. An alternative is a two-country IS/LM model with exchange rate dynamics added. Its dynamic properties under Uncovered Interest Rate Parity are briefly explored. However, this problem is much deeper and beyond the scope of this technical paper. The future direction of research is suggested in the last part of paper.

Key words: Portfolio Balance, Mundell–Fleming, UIP,
JEL classifications: F32, F41

1. Introduction

For decades, a central doctrine of open macroeconomics has been that the portfolio balance and Mundell–Fleming (or IS/LM/BP) models — the familiar analytical tools of international finance for policy purposes — both contain three independent equations that determine three variables: the domestic and foreign interest rates and the spot exchange rate.

This paper tries to demonstrate that this concept seems to be incorrect from general equilibrium perspective. If economic actors satisfy the balance sheet and the portfolio allocation restrictions, then setting the exchange rate is beyond the models' reach. The key reason why is that the domestic and foreign economies' nominal net foreign asset holdings are constant in the short run. Not only one country's external assets (the other's external liabilities), but at a given exchange rate their net positions can only change over time through the accumulation of its current account surpluses or deficits. In the temporary equilibrium, each country's net foreign assts constraint forces its money market to clear when its bond market is satisfied and vice-versa.

　　The arguments supporting these findings proceed in the Keynesian models concerned. In section 2, the stock and flow of accounts in a two-country macro-economy is analyzed. In section 3, the Portfolio balance, Mundell–Fleming, and Uncovered Interest Rate Parity exchange rate theories are analyzed. In section 4, the inability to settle the exchange rate in Portfolio Balance model will be demonstrated. In section 5, the same result in Mundell–Fleming model will be demonstrated. The independent temporary equilibrium conditions that exist boil down to linked IS/LM models for the two countries, with activity levels and interest rates (X and X^* and i and i^* respectively) as the endogenous variables. Comparative static exercises become of interest, taking the spot exchange rate (e), its expected change over time (ε), and other variables as pre-determined. In section 6, examples for a small country will be presented. In section 7, the dynamic extensions will be tried and some additional considerations will be shown. Before proceed to content, three methodological observations are worth adding. First, although no one seems to have noted it in the standard models, the irrelevance of the exchange rate to temporary equilibrium in open economy asset markets has been pointed out in other contexts, e.g. in a two-country growth model by Foley and Sidrauski (1971), an overlapping generations model by Kareken and Wallace (1981), and a computable general equilibrium model by Rosensweig and Taylor (1990). Second, intertemporal optimization as surveyed by Obstfeld and Rogoff (1995) could be added to the analysis without altering the tenor of the results. In temporary equilibrium, intertemporal models incorporate portfolio balances and a real side macro model. Over time, they use Euler differential equations to determine the exchange rate under the Uncovered Interest Rate Parity and consumption levels by using optimal saving rules formulated by Ramsey (1928). The Euler equations determine changes in consumption levels and the exchange rate by intertemporal arbitrage. They are not directly relevant to the discussion of temporary equilibrium in sections 4–6, which focuses on levels of stocks and flows. In section 7, the inconsistency that the traditional interpretation of the portfolio balance model with the intertemporal arbitrage that assumes Uncovered Interest Rate Parity. Finally, the results here help explain why an enormous amount of empirical literature finds that "fundamentals-based" models of the exchange rate have little predictive power. If the standard models cannot determine the spot rate, then econometric tests based on them would almost likely to fail.

2. The Accounting Matrix Sheet

　　One of the surest ways to tackle the economic problem is to examine accounting. Table 1 and Table 2 attempt this task for international trade and financial relations. It is unfortunately true that full accounting in models containing real and financial sectors requires a wagonload of symbols. The details in both tables are needed to demonstrate the results that follow. They follow broadly from a scheme proposed by Godley (1996).

　　Table 1 presents the domestic and foreign countries' flow accounts in the form of the spreadsheet of accounting matrix. Table 2 presents the corresponding balance sheets. Since the emphasis is on external transactions, economic actors in each country are aggregated into just three groups — a private sector comprising households and non-financial business sector, government sector, and

Table 1-1　The two-country accounting matrix of trade, interest flows and capital movements

	Current Domestic Expenditure				Domestic External Receipt			Changes in Domestic Claims			Totals	Exchange Conversion	
	Output Cost (1)	Private (2)	Gover't (3)	Banks (4)	Capital formation (5)	Exports (6)	Foreign interest (7)	Money (8)	Domestic Bonds (9)	Foreign Bonds (10)	(11)	Foreign to Home	Domestic to Foreign
Output Incomes (A)		PC	PG		PI	Pa^*X^*					PX		
Private (B)	V		iT_h	Π_b			eiT^*_h				Y_h		
Gover't (C)		τY_h									Y_g		
Banks (D)			iT_b				eiR^*				Y_b		
Flows of Funds													
Private (E)		S_h						$-\dot{M}$	$-\dot{T}_h$	$-e\dot{T}^*_h$			
Gover't (F)			S_g						$\dot{T}$		0		
Banks (G)				0				$\dot{M}$	$-\dot{T}_b$	$-e\dot{R}^*$	0		
External													
Imports (H)	eP^*aX											e	
Exports (I)						$-Pa^*X^*$							$1/e$
Int. in (J)							$-ei^*T^*_{ext}$					e	
Int. out (K)			iT_{ext}										
Change in ext. liability (L)									$-\dot{T}_{ext}$				$1/e$
Change in ext. assets (M)										$e\dot{T}^*_{ext}$			$1/e$
Total (N)	PX	Y_h	Y_g	Y_b	0	0	0	0	0	0			

Table 1-2　The two-country accounting matrix of trade, interest flows and capital movements

	Current Domestic Expenditure				Domestic External Receipt			Changes in Domestic Claims			Totals
	Output Cost	Private	Gover't	Banks	Capital formation	Exports	Foreign interest	Money	Foreign Bonds	Domestic Bonds	Totals
	(1*)	(2*)	(3*)	(4*)	(5*)	(6*)	(7*)	(8*)	(9*)	(10*)	(11*)
Output Incomes (A*)		P^*C^*	P^*G^*	Π^*_b	P^*I^*	P^*aX					P^*X^*
Private (B*)	V^*		$i^*T^*_f$				iT_f/e				Y^*_f
Gover't (C*)		Y^*_f									Y^*_g
Banks (D*)			$i^*T^*_b$				iR/e				Y^*_b
Flows of Funds											
Private (E*)		S^*_f						$-\dot{M}^*$	$-\dot{T}_f/e$		0
Gover't (F*)			S^*_g						$\dot{T}^*$		0
Banks (G*)				0				$\dot{M}^*$	$-\dot{T}^*_b$		0
External											
Imports (H*)											
Exports (I*)	Pa^*X^*/e				$-P^*aX$						
Int. in (J*)			$i^*T^*_{ext}$			$-iT_{ext}/e$					
Int. out (K*)					0	0	0				
Change in ext. assets (L*)					0	0	0				
Change in ext. liability (M*)									$-\dot{T}^*_{ext}$		
Total (N*)	P^*X^*	Y^*_f	Y^*_g	Y^*_b	0	0	0	0			

Table 2 Balance sheet for two countries

Domestic Country					Foreign Country				
Private Sector		Banking Sector		Government Sector	Private Sector		Banking Sector		Government Sector
M	Ω	T_b	M	T	M^*	Ω^*	T_b^*	M^*	T^*
T_h		eR^*			T_f^*		R/e		
T_h^*					T_f/e				
qPK					$q^*P^*K^*$				

banking sector which consolidates commercial banks and central banks. Variables referring to the home and foreign private sectors are labeled with subscripts h and f respectively. The foreign country's stocks and flows are denoted by asterisks in Table 1 and Table 2.

As shown in Table 2, the primary assets underlying the flow accounts in the accounting matrix are capital stocks valued at their asset prices (qPK and $q^*P^*K^*$ where q and q^* are "valuation ratios" or levels of "Tobin's q") and outstanding short-term bonds or T-bills issued by the two governments (T and T^*, with interest rates i and i^* respectively). Both banking sectors carry a subscript b, but they are distinguished by asset portfolio. The domestic sector's assets are domestic bonds T_b and foreign bonds as reserves with a value eR. The foreign sector holds foreign bonds T_b^* and domestic bonds as reserves with a domestic value R/e. The banks have zero net worth and their liabilities are the money supplies M and M^* respectively. In their portfolios, the private sectors hold the relevant capital stock qPK or $q^*P^*K^*$, both include bonds and the domestic form of money M or $M^{*\,1)}$. In the accounting matrix, the two main accounting principles are that corresponding row and column totals are equal, and that in each country the entries in each row are valued at the same price. The accounts for the domestic country are shown to the left, and for the foreign to the right. Cross-border flows are mediated by "exchange conversions" between the two sides (the conversions also involve a sign change because an inflow to one country is an outflow from the other). The spot exchange rate, e, is the price of domestic currency to foreign currency.

Output and income generation relationships are shown in the northwest of each country's section of the accounting matrix. Real output levels are X and X^* in the domestic and foreign countries respectively, with prices P and P^*. The corresponding costs of production are broken down in columns (1) and (1^*). Total values of output PX and P^*X^* include payments for nominal value added (V and V^*) and imports (eP^*aX and Pa^*X^*/e). In both countries, imports are assumed to be input for domestic and foreign sales, thus becoming elements of cost[2]. Real imports are scaled to outputs by coefficients a and a^*, which could depend on relative prices such as the real exchange rate $e\,P^*/P$ in the neoclassical models.

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1) The private sector in each country is assumed to hold only the domestic money in rather rough way. It would be more accurate to deal the general case by using more symbols, but the accounting matrix already has many symbols.

2) Other specifications such as treating imports as negative exports or additional components of consumption demand are possible, but would not change the main results of this paper.

Rows (A) and (A*) show that outputs are used for the usual purposes — private consumption, government expenditure, private investment, and exports. For future reference, note how exports in cells (A, 6) and (A*, 6*) are valued in domestic prices. The row and column totals in (A) and (1) and (A*) and (1*) are the same, in line with the accounting convention noted above.

Rows (B)–(D) and (B*)–(D*) show how incomes of the private sectors, governments, and banks are generated; their outlays and levels of saving appear in columns (2)–(4) and (2*)–(4*). In rows (B) and (B*), the private sectors receive incomes from value-added and interest payments on their holdings of domestic and foreign bonds (for the domestic private sector, the receipts are iT_h and $ei^*T_h^*$ respectively). The simplest way to deal with interest incomes on bank assets is to assume that they are passed to the private sectors. In row (D), for example, the domestic banking sector has interest income $Y_b = iT_B + ei^*R^*$. It transfers these returns Π_b to the private sector in column (4): $\Pi_b = Y_b$. As a result of such transfers, savings of the banking systems are equal to zero in cells (G, 4) and (G*, 4*). The governments get their revenues from taxes on private incomes (τY_h and $\tau^* Y_f^*$) in rows (C) and (C*).

Rows (E)–(G) and (E*)–(G*) show the different sectors' flows of funds. The accounting convention is the sources of funds (=saving and increases in liabilities) are positive and uses of funds (=increases in assets) are negative. The equation for the domestic private sector in row (E),

$$S_h - PI - \dot{M} - T_h - e\dot{T}_h^* = 0,$$

shows that it uses its saving S_h for capital formation PI and increased holdings of domestic money $\dot{M}$, domestic bonds $\dot{T}_h$, and foreign bonds valued by the exchange rate $e\dot{T}_h^*$. (The (·) above each variable stands for its change over time)). Combined with differentiation of the domestic household balance sheet in Table 2, this flow of funds equation gives the change in nominal domestic private sector wealth as

$$\dot{\Omega} = S_h + \dot{e}T_h^* + (\dot{q}P + q\dot{P})\,K,$$

i.e. $\dot{\Omega}$ is the sum of saving and capital gains (or losses) from changing asset prices.

Row (F) shows that domestic government saving S_g must be negative if it is issuing a positive flow of new bonds ($S_g = -\dot{T} < 0$ when $\dot{T} > 0$). In row (G), the fact that the banking sector's saving has been set to zero means that the growth of the money supply responds only to changes in bank assets, $\dot{M} = \dot{T}_b + e\dot{R}^*$.

Three international payments flows go in each direction, for a total of six rows (H)–(M) and (H*)–(M*). In terms of their domestic prices, exports of Pa^*X^* and P^*aX in cells (A, 6) and (A*, 6*).

Between rows (I)–(I*) and (H*)–(H), domestic and foreign exports are converted to the other country's prices by the inverse of the exchange rate (1/e) and its level (e) respectively and then become imports in columns (1*) and (1).

Second, in column (7) the domestic private sector and banks hold foreign bonds in quantities T_h^* and R^*. The values of their interest receipts in domestic prices are $ei^*T_h^*$ and ei^*R^*. With $T_{ext} = T_h^* + R^*$ as domestic gross foreign assets, its total interest income is ei^*T_{ext} in cell (J, 7). After an exchange

conversion between rows (J) and (J*), the foreign government's interest payments in domestic prices on its bonds held abroad count as a fiscal outlay $i^*T^*_{ext}$ in cell (J*, 3*). The domestic government's interest payments on its gross external liabilities $T_{ext} = T_f + R$ are treated analogously in column (7*), rows (K*)–(K), and column (3).

Finally, foreign asset holdings change over time, for instance, $\dot{T}_{ext} = \dot{T}_f + \dot{R}$ is the equation for foreign accumulation of the domestic government's bonds at domestic prices. The exchange conversion is between rows (L) and (L*), and column (10*) gives bond accumulation in foreign prices. The transactions shown in the accounting matrix correspond to the usual categories in balance of payments accounts, i.e. trade in goods and services, factor payments, and capital movements. In a formal model, the trade flows would be derived from activity levels and relative prices, and interest rates would adjust to make sure that asset markets (incorporating both foreign and domestic flows) clear. Interest rates on asset stocks would set the levels of factor payments.

As Godley (1996) emphasizes, a puzzle in the accounting matrix is that while it contains numerous international transactions, there are no "balance of payments" in it. It is not obvious why all the cross-border flows with their exchange conversions should add to the overriding balance, especially since all have their own separate determinants. But the standard accounting does make sense. To see why, it is helpful to think in terms of net foreign assets N of the domestic country, which can be defined as

$$N = e(T^*_h + R\,) - (T_f + R) = eT^*_{ext} - T_{ext} \qquad (1)$$

or domestic holdings of foreign bonds valued at the spot exchange rate minus the value of its own bonds held abroad. In (1) it is clear that N follows from historical gross asset and liability positions, i.e. its level is set by the country's history of current account deficits and surpluses and the ways in which they were financed. It is shown below that net foreign assets cannot jump in unconstrained way in temporary equilibrium. Any change in the level of gross assets has to be met by an equal change in gross liabilities to hold N left unchanged. In this way, (1) becomes a binding constraint on macroeconomic adjustment.

Net foreign assets can take either sign (including holdings of equity, they were over — $2 trillion for the U.S. at century's end). As nominal terms, N and its foreign counterpart — N/e are subject to capital gains and losses due to movements in the exchange rate. The quantity changes in N are discussed below. Summing and substitutions among the rows and columns of the accounting matrix give the following chain of equalities:

$$S_h + S_g - PI = e(\dot{T}^*_h + \dot{R}^*) - (\dot{T}_f + \dot{R}) = \dot{N}$$
$$= [Pa^*X^* + ei^*(T^*_h + R^*)] - [eP^*aX + i(T_f + R)] = -S_f \qquad (2)$$

where S_f stands for the domestic country's "foreign savings" or current account deficit (if the domestic country is saving less than it invests, then the rest of the world must be providing savings to finance the shortfall).

In the first line of (2) the sum of domestic sources of savings minus investment is equal to the

increase in net foreign assets. In turn, in the second line $\dot{N}$ is equal to the surplus on current account or $-S_f$. All presentations of an economy's balance of payments are rearranged to equations like those in (2). What they are basically saying is that (apart from capital gains and losses), net foreign assets change over time in response to the current account. Decisions about how net assets are accumulated in terms of portfolio allocations (including foreign reserves) are discussed below.

3. Exchange Rate Determination

Three standard exchange rate models — portfolio balance, Mundell–Fleming, and UIP in which a floating exchange rate is freely adjusted over time (sometimes with discontinuous jumps) are the focus of attention.

UIP emerges from intertemporal foreign currency arbitrage as analyzed in the 1920s by Keynes (1923), among others. It gives a rule by which the exchange rate can be determined as an asset price from expected changes in its value over time. UIP is intrinsically dynamic, because it is based on arbitrage of own rates of return over time.

If ε is the expected short-term change in the spot rate e, the UIP rule can be written as

$$e = \varepsilon/(i - i^*) \tag{3}$$

The current exchange rate should be equal to its expected change, capitalized by the difference between the two interest rates, $i - i^{*}$[3]. In line with Keynes's (1936) predilections in Chapter 17 of "The General Theory", (3) can be restated in the form of an own-rate of interest after a simple approximation that $(\varepsilon/e)\, i^* \approx 0$ on the right-hand side,

$$i = i^* + (\varepsilon/e)(1 + i^*) \approx i^* + \varepsilon/e \tag{4}$$

That is, the foreign interest rate will exceed the domestic rate whenever the own currency is generally expected to depreciate. If $\varepsilon > 0$, a Japanese investor going into dollars anticipating that the yen-dollar spot rate will be depreciated and has to be compensated the U.S. rate i^* that exceeds i. The "spread" between the interest rates will become greater as ε/e, the expected relative change in the exchange rate, rises.

In contrast with UIP's emphasis on expectations about spot exchange rates in the future, macro level theories concentrate on the exchange rate's linkages with aggregates such as the trade balance, the composition of asset portfolios, or the overall balance of payments.

A floating rate is supposed to converge at a level that "clears" macro balances. With the rate

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3) For the moment, ε can be interpreted in a Keynesian sense as summarizing the market's perceptions or views about how the exchange rate is likely to shift over time; a more contemporary interpretation in terms of myopic perfect foresight appears in section 7 below. Also note that ε stands for an absolute as opposed to a percentage change per unit of time in e.

fixed, the balances may not achieve equilibrium in well-determined ways.

The two Keynesian models incorporating the financial side of the balance of payments have been accepted widely from the 1960s and 1970s[4]. One model concentrates on "portfolio balances," and claims that the exchange rate along with the two bond interest rates is determined by equilibrium conditions in three of four relevant financial markets — for domestic and foreign moneys M and M^* and bonds T and T^* (the fourth market is supposed to clear by Walras's Law). It is shown below that this claim would be incorrect because there is just one independently clearing asset market in each country.

The other model is usually attributed to Mundell (1963) and Fleming (1962). In an open economy described by a (3×3) system of equations, adjustment dynamics are based on the ideas that the output level responds to excess demand of goods from an IS relationship and interest rate shifts in response to asset market imbalances in a LM. The exchange rate is assumed to adjust when the balance of payments does not clear. On the other hand, if the exchange rate is pegged then international reserves have to be the adjusting variable, making monetary policy endogenous. Solving all three equations simultaneously gives the usual stability and comparative static results.

The Mundell–Fleming "duality" between reserves and the exchange rate evidently presupposes that a balance of payments exists, with a potential disequilibrium that has to be cleared. But as shown in connection with Table 1, the balance of payments is at most an accumulation rule for net foreign assets and has no independent status as an equilibrium condition. The Mundell–Fleming duality seems to be irrelevant, and in temporary equilibrium the exchange rate does not depend on how a country operates its monetary (especially foreign reserve policy).

The assessment of the portfolio balance and Mundell–Fleming models suggests that they are not satisfactory approaches to exchange rate determination. In contemporary markets it appears that the rate is extrinsic to macro equilibrium as it emerges from adjustments in variables such as interest rates or the level of economic activity. A floating exchange rate is not a price that equilibrates markets. Apart from the markets in which its own future values are set through UIP or other intertemporal behavioral practices.

Evidently, dynamic considerations have to be incorporated. Though it does not fit the data (Blecker, 2002), UIP is the typical intertemporal model to consider. A formulation incorporating IS,

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4) The macro relationships that must be considered in the contemporary world relate to both current accounts and capital accounts. Before and in the first decades after World War II, it made sense to concentrate only on the trade account. A model proposed 40 years ago by Salter (1959) and Swan (1960) contained price ratio of tradable and non-tradable goods. As far as trade is concerned, the exchange rate can be interpreted as becoming increasingly over-valued when this internal price ratio falls. But for the industrialized countries at least, payments related to trade are now such a small share of total external transactions that the model is obsolete (Eatwell and Taylor, 2000). In turn, Salter–Swan was the culmination of a long string of models of the trade account, which assumed that capital flows were exogenous. They included an elasticity approach ("Marshall–Lerner conditions" on trade elasticity), an absorption approach, and analysis of internal vs. external balance that led to Salter–Swan. A monetary approach to the balance of payments worked out in the 1950s and 1960s fed into the portfolio balance and Dornbusch models discussed below, as well as many econometric attempts to predict shits in the spot rate (section 7).

LM, and UIP is presented in section 7. It generates cyclical dynamics, as opposed to the saddle path jumps characteristic of recently popular dynamic optimization models mentioned above.

4. The Portfolio Balance Model

The portfolio balance model was introduced as an extension of Tobin's (1969) financial market analysis from closed to open economy macroeconomics. The basic idea was that a floating exchange rate should be determined by some contemporary market clearing mechanism, the message of surveys such as those by Branson and Henderson (1985) and Isard (1995). In this section, why this plausible notion fails will be demonstrated. In other words, if it is not fixed by the authorities, the exchange rate is determined by forces beyond those contained in a temporary equilibrium asset allocation model. The basic assumptions in this section are as follows.

Private sectors and banking sectors at home and abroad are the only actors holding financial assets. They take the form of national money supplies and short-term government bonds that pay domestic and foreign interest rates i and i^* respectively. The money supplies are backed by domestic and foreign bonds held by the two banking sectors[5].

Both private sectors and banks satisfy their balance sheet restrictions, i.e. the total values of their assets are always equal to the total values of their liabilities plus net worth.

Apart from capital gains and losses induced by jumps in the exchange rate, total net foreign assets N (or –N/e) held by banks and the private sector in each country are constant in the short run. The reason is already clear from equation (2), which shows that N can only change over time in response to a surplus or deficit of current account.

A portfolio balance model is assumed to re-equilibrate in the short run to shocks such as operations of the monetary authorities and exogenous shifts in asset preferences. In the new temporary equilibrium, portfolio compositions may shift. In line with their key role in clearing asset markets, domestic and foreign interest rates are taken as the main endogenous variables.

Under these hypotheses, it will be shown that if the two markets for bonds clear, then so will the two markets for domestic moneys and vice versa. There are just two independent asset market equilibrium conditions in the system.

Traditionally, portfolio balance models are formulated to deal with only the financial side of an open economy. Following this practice, capital stocks are ignored in this section. In the domestic banking sector, the stock of M changes with open market operations in domestic bonds and shifts in the level of reserves. That is, M responds to its asset base as intended by banks. Notation to represent such interventions is introduced below. Many presentations treat banking sector liabilities as predetermined. But since T_b and R^* can jump in the short run, just setting M instead of considering shifts in its underlying assets may mislead the analysis.

For algebraic convenience, asset holdings are set as shares of private sector wealth levels Ω and

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5) The central bank is assumed to intervene for control its holdings of assets. A Post-Keynesian scenario in which interest rates are set exogenously and money supplies adjust would be straightforward to work through.

Ω^*. The shares can depend on interest rates, wealth levels themselves, the level of economic activity, the exchange rate, its expected change, and other variables. The domestic excess demand and supply functions can be written as

$$\mu\Omega - M = 0 \tag{5}$$
$$T_h - \eta\Omega = 0 \tag{6}$$
$$eT_h^* - \phi\Omega = 0 \tag{7}$$

Similarly, asset balance equations for the foreign private sector are

$$\Omega^* - M^* = 0 \tag{5^*}$$
$$T_f/e - \eta^*\Omega^* = 0 \tag{6^*}$$
$$T_f^*/e - \phi^*\Omega^* = 0 \tag{7^*}$$

If the private sectors respect their balance sheets, the demand proportions must satisfy the restrictions $\mu + \eta + \varphi = 1$ and $\mu^* + \eta^* + \phi^* = 1$. There are four asset market equilibrium conditions. Two stand for that excess demands for the money disappear in (5) and (5^*). The others state excess supplies for the two flavors of bonds equal to zero,

$$T - T_h - T_f - T_b - R = -\eta\Omega - e\eta^*\Omega^* - T_b - R = 0 \tag{8}$$
$$T^* - T_h^* - T_f^* - T_b^* - R^* = T^* - \phi\Omega/e - \phi^*\Omega^* - T_b^* - R^* = 0 \tag{8^*}$$

Finally, as noted above the domestic net foreign assets N are defined by equation (1). To explore the implications, domestic gross external assets and liabilities can be regarded as already defined in connection with Table 1 and Table 2, $T_{ext}^* = T_h^* + R^*$ and $T_{ext} = T_f + R$. Then N is set in the point-slope representation of (1) in Figure 1. Its controlling variables are the exchange rate and historically given levels $\overline{T}_{ext}^*$ and $\overline{T}_{ext}$ of external claims. The exchange rate changes generate capital gains or losses in N. Depreciation or a higher value of e rotates the "external assets" line representing (1) counter-clockwise around the ($\overline{T}_{ext}^*$, $\overline{T}_{ext}$) point, increases domestic net foreign assets in domestic (N) and foreign (N/e) currency terms[6]. The line also constrains external asset positions when they jump away from their initial values if the model's temporary equilibrium is perturbed. The totals T_{ext}^* and T_{ext} have to rise or fall together to hold N constant. This simultaneous increase in a country's foreign assets and liabilities is directly analogous to a firm increase deposit to a bank from which it takes a loan, a ubiquitous practice in most lending operations. Because N is defined by e, $\overline{T}_{ext}^*$, $\overline{T}_{ext}$, in (1), the equation cannot surely be solved for the exchange rate (Fig. 1).

Uniformly in the literature, the portfolio balance model has been set up with the balance sheet identities $\Omega = M + T_h + eT_h^*$ and $\Omega^* = M^* + T_f/e + T_f^*$ used to define the levels of wealth. Then Ω and Ω^* are plugged into asset market balances which are solved for the interest and exchange rates.

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6) The diagram presupposes that the domestic country is a net creditor at the ruling exchange rate. To get to the net debtor case, the external assets schedule can be rotated.
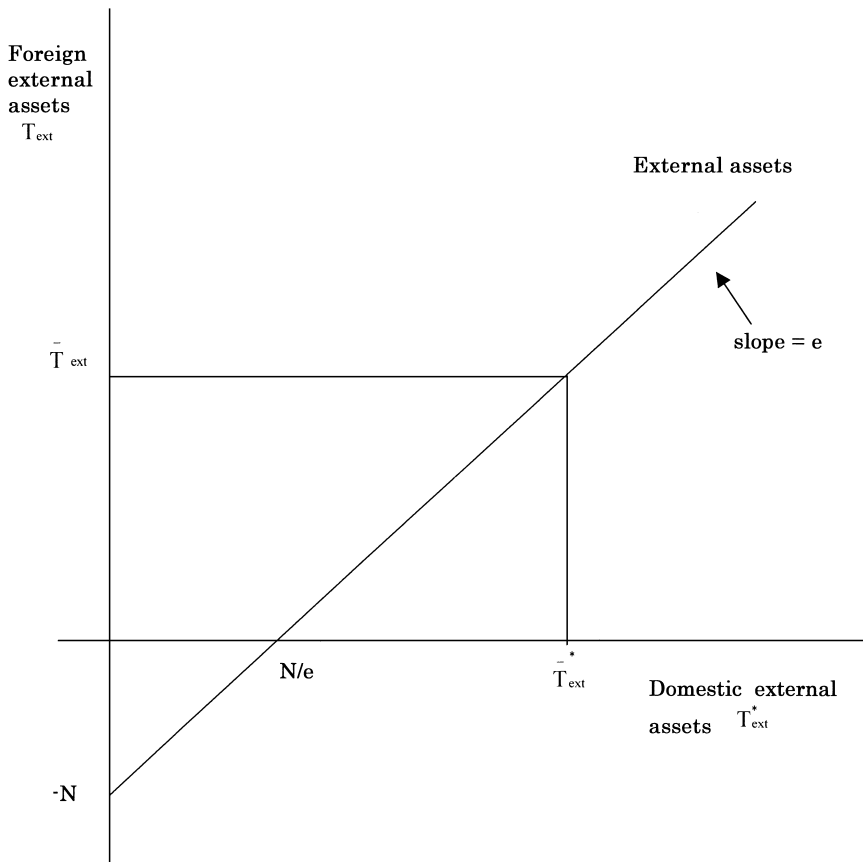
Fig. 1　Determination of net foreign assets from initial external assets

This algorithm makes sense insofar as private wealth is predetermined at any time by a history of capital gains and saving flows (instantaneous capital gains due to a contemporary exchange rate movement can also be taken into account). But the standard formulation leaves aside the fact that asset holdings of the private sectors are not fully free to vary. Besides Walras's Law, they are constrained by the balance sheets of the banking sector and net foreign assets constraint. These restrictions make dependent two and not just one of the market equilibrium conditions (5), (5*), (8), and (8*), increasing from zero to one the degrees of freedom available to the underlying triple of variables i, i^*, and e.

　One way to incorporate balance sheet restrictions into Tobin-style models is to express the wealth levels Ω and Ω^* in the market balance equations in terms of national primary assets[7]. Walras's Law takes a step in this direction. It can be written as

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7) Another way is to linearize three asset market balance equations around an initial equilibrium and also to solve them for "small" changes in i, i^*, and e, subject to the accounting restrictions mentioned in the paper. In this case, the Jacobian matrix of this system will be singular.

$$(M^* - \mu^*\Omega^*) + (T - \eta\Omega - e\eta^*\Omega^* - T_b - R) + e\,[T^* - (\phi\Omega/e) - \phi^*\Omega^* - T_b^* - R^*] = 0$$

The banking sectors satisfy their balance sheets and the "adding up" restrictions $\mu + \eta + \varphi = 1$ and $\mu^* + \eta^* + \phi^* = 1$ on portfolio allocations apply, this equation reduces to

$$\Omega + e\Omega^* = eT^* \tag{9}$$

or worldwide wealth is equal to the value of outstanding government debt.

Equation (9) is familiar but not immediately helpful since it does not pin levels of national wealth. To determine Ω and Ω^* explicitly it suffices to assume that either (8) or (8*) holds, or that at least one bond market clears. Along with the assumption that there are no black holes in balance sheets, if (8) holds, the balance sheet for the domestic private sector is written in the forms

$$\Omega = M + T_h + eT_h^* = (T_b + eR^*) + (T - T_f - T_b - R) + eT_h^* = T + e(R^* + T_h^*) - (T_f + R)$$

or from (1),

$$\Omega = T + N \tag{10}$$

Substitution into Walras's Law (9) gives

$$\Omega^* = T^* - N/e \tag{10*}$$

Each nation's wealth is made up of its outstanding government debt plus its net foreign assets. By shifting the values of N and N/e, changes in the exchange rate affect Ω and Ω^*. The devaluation raises home wealth and reduces foreign wealth.

Now we can use (10) and (10*) to show that equations (5) and (8) for money and bond market balance in the domestic country are equivalent (similar calculations work for the foreign country as well). With (10) setting Ω, equation (5) for money demand-supply balance becomes

$$\mu(T + N) = M = T_b + eR^* \tag{11}$$

Using (10) and (10*), equation (8) for the bond market can be written as

$$\eta(T + N) + e\eta^*(T^* - N/e) = T - T_b - R \tag{12}$$

Formulas (11) and (12) superficially look different, but a few quick substitutions show that they are the same. To get (11) from (12), substitute for R from (1), rearrange the resulting expression, and impose the condition $\mu + \eta + \phi = 1$. This result coincides the accepted finding is that in a closed economy if the bond market clears then so does the money market. The net foreign asset constraint is the bridge that allows this reasoning to be extended to a two-country capital market.

As already noted, equation (1) enters the system as a binding restriction on spot transactions in external securities. Consider a shift in foreign preferences toward domestic bonds, so that T_f and T_{ext} jump up. If the foreign country's reserves R stay constant, then some element in the term $e(T_h^*$ $+ R^*) = eT_{ext}^*$ has to jump up as well. The obvious candidate is R^*. To acquire more domestic bonds, the foreign private sector must transfer foreign bonds that it holds across the border, valued at the spot rate e. They will immediately appear in domestic foreign reserves, as T_{ext}^* and T_{ext} shift up from their initial values $\overline{T}_{ext}^*$ and $\overline{T}_{ext}$ along the external assets line in Figure 1. More reserves immediately push up the domestic money supply. Empirically, reserve upswings after capital inflows that lead to growth of the money supply are frequently observed. Since the Latin American crises in 1980s, they have been a familiar precursor to emerging market debt cycles headed by surging capital inflows.

Although recent experience underlines the practical difficulties in principle such a monetary expansion can be controlled. This observation is likely to arouse contradictions such as the equivalence of each country's two asset market balances, but sought to preserve the traditional portfolio balance model by excluding reserve changes. For example, the exchange rate could float and each country's central bank could control both components of its monetary base. The exchange rate would adjust so as to keep net foreign assets N constant. This is the outcome under flexible rates. Note that it determines perfectly the current exchange rate.

There seems to be at least two errors in this argument. One is that the banking sectors have tools at its disposal to control both components T_b and eR^* of the money supply with N constant and no need for a floating rate. The other problem is that if one simply postulates the constant reserves without specifying what the banking sector does to hold them steady, then an exchange rate that varies to satisfy the net foreign assets constraint generates implausible results.

To see how the domestic banking sectors can control its asset position, suppose that the interest rates i and i^* adjust to clear the excess supply functions for domestic and foreign bonds. These relationships shift in response to changes in the spot exchange rate (through substitution effects and its wealth effects on N, –N/e, Ω and Ω^*), the expected change in the exchange rate (through substitution effects), levels of wealth and output, etc. Continuing with the example above, assume that the foreign demand functions for foreign and domestic bonds shift from $\phi^*\Omega^*$ and $\eta^*\Omega^*$ to $\phi^*\Omega^*$ $- \Delta^*$ and $\eta^*\Omega^* + \Delta^*$ respectively. The capital inflow makes its reserves and money supply increase by $e\Delta^*$. It is well known (Isard, 1995) that central bank can counter such a shock to portfolio holdings in at least two ways. It can offset the reserve increase by selling a quantity $e\Gamma^*$ of foreign bonds and using the proceeds to buy domestic bonds (with the help of the foreign central bank), and reverse the monetary expansion by selling a quantity Λ of domestic bonds in the open market operation.

After these portfolio adjustments, the home bond market balance can be written as

$$T - [\overline{T}_b + e\Gamma^* - \Lambda] - R - \eta - e(\eta^*\Omega^* + \Lambda^*) = 0 \qquad (8a)$$

where $\overline{T}_b$ stands for the initial level of banking sector holdings of domestic bonds and $[\overline{T}_b + e\Gamma^* - \Lambda]$ is the level after the interventions, and the foreign balance as

$$e(T^* - T_b^* - R^*) - \phi\Omega - e(\phi^*\Omega^* - \Lambda^*) = 0 \qquad (8a^*)$$

from (1) extended to include Λ^* and Γ^*, the new level of reserves is

$$eR^* = N - \phi\Omega + R + e(\eta^*\Omega^* + \Lambda^*) - e\Gamma^* \qquad (1a)$$

To hold domestic banks' bond stock at $\overline{T}_b$, the central banks can sterilize the effects of their foreign bond sale by setting $\Lambda = e\Gamma^*$ in the bracketed term on the left-hand side of (8a). Then after a substitution from (1a) into ($8a^*$) to remove terms in eR^*, the simultaneous equation is obtained.

$$T - \overline{T}_b - R - e\Lambda^* - \eta\Omega - e\eta^*\Omega^* = 0 \qquad (13)$$

and from (10^*)

$$e(T^* - T_b^*) - N - R - e\Gamma^* - e(\phi^* - \eta^*)\Omega^* = 0 \qquad (13^*)$$

Assuming existence conditions are satisfied, for any value of e (13) and (13^*) will solve for i and i^* as functions of Λ^* and Γ^*. Plugging the interest rate solutions into ϕ and η^* in (1a) gives

$$eR^* = eR^* = N + R + f(\Lambda^*, \Gamma^*) + e(\Lambda^* - \Gamma^*) \qquad (1b)$$

The function $f(\Lambda^*, \Gamma^*)$ denote the amount by which Γ^* would have to differ from Λ^* to hold R^* to its initial value. Although practical applications could prove difficult, (1b) shows that for a given Λ^* the central bank can use Γ^* to steer R^* to the level it desires. Net foreign assets stay constant and bond markets clear through changing interest rates, with no need for e to be an endogenous variable "dual" to the policy-determined stock of reserves[8].

Of course, one might simply postulate that reserves do not change, without taking into consideration tools such as Λ and Γ^* that the central banks can use to make this situation come about (this was the theoretical stance taken by Mundell and Fleming in formulating their model, which the carried over to portfolio balance). It might then look reasonable to assume that e adjusts to hold N constant in (1) if the system is disturbed. But there are some problems.

One is that in the real world (as opposed to optimal growth models in which asset prices can jump to hold net worth constant), it is hard to find cases in which wealth determines the values of its components, especially in the short run. The nominal net worth of a household, firm, nation, or the world is determined by its real asset positions and the relevant asset prices. For individual players their net worth does not determine asset valuations, but causality runs the other way.

Further, in empirical practice, (1) or (1a) would not be a appropriate "third equation" for the

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8) The interest rate changes could induce shifts in the exchange rate over time. On the basis of the comparative static results below, it seems likely that the compensated capital inflow discussed in the text will make i fall and i^* rise. Under UIP and myopic perfect foresight (section 6), the exchange rate would tend to appreciate $\dot{e}$.
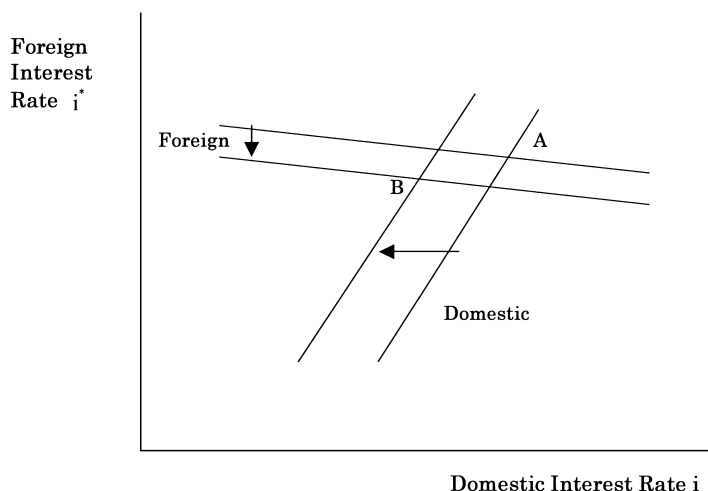
Foreign
Interest
Rate i[*]

Foreign

A

B

Domestic

Domestic Interest Rate i

Fig. **2**　The effect of depreciation of domestic currency
Note: Domestic monetary expansion is similar, except that only the domestic schedule shift.

exchange rate because the impact of a jump in T_f (with the other variables in the equation held constant) would increase the value of e. This depreciation could reverse if portfolio compositions shift strongly with e, or when feedback through the bond markets is taken into account. However, it is disturbing. Capital inflows are supposed to strengthen, not weaken, the domestic currency. The portfolio balance model, traditionally interpreted, gives the expected appreciation. If the two interest rates varied to clear each country's money market (with reserve levels and money supplies held constant by assumption), then the third equation could be the domestic country bond balance (8) or (8a). Under assumptions discussed below, one would have $\partial\eta/\partial e > 0$ and $\partial(e\eta^*)/\partial e > 0$, so absent a strong wealth effect through Ω^*, e would decline in response to an exogenous portfolio shift as discussed above. Trying to save the model by replacing dependent equation (8) with independent (1) subverts its original intent.

　To close the argument, it makes sense to work through the short-run comparative static implications of the equilibrium conditions (13) and (13[*]), with domestic and foreign interest rates as the endogenous variables. In domestic financial markets, changes in i and i[*] are usually assumed to have effects with opposite signs. A higher level of i will reduce excess demand for domestic money and excess supply of domestic bonds, with a higher i[*] working the other way. If domestic and foreign bonds are close substitutes in (8a) or (13), then the domestic bond market schedule in Figure 2 will have a slope of a bit more than 45 degrees in Figure 2.

　With the effects of the net foreign asset constraint in the foreign bond market taken into account in (13[*]), interest rate effects are likely to have the same, negative sign. In (13[*]), $(\phi^* + \eta^*)\Omega^* = (1 - \mu^*)\Omega^*$ and presumably foreign money demand $\mu^*\Omega^*$ declines with increases in both i and i[*]. For a small country, the foreign schedule representing (13[*]) will have a slightly negative slope if changes in i have minor effects on i.

　Obvious comparative static shifts to consider are an expansionary open market operation (with the

central bank buying domestic bonds), a capital inflow and an exchange of foreign for domestic bonds as discussed above, and movements in the expected and current exchange rates.

In (8a), an open market bond purchase (a negative Λ) reduces the left-hand side, forcing i to decline. The domestic schedule shifts left, reducing the domestic rate and raising the foreign rate. The bond swap (a positive Γ^*) shifts the domestic schedule rightward and the foreign schedule up. The domestic rate rises. The shift in the foreign rate is ambiguous, but if $e\Gamma^*$ is offset by an equal Λ^* in (8a), i^* will rise because the domestic schedule does not shift. A capital inflow Δ^* compensated by these shifts the domestic schedule left. When more external demand for domestic bonds push up their price, they force a lower asset return i and higher i^*. To offset the lower rate, the central bank would have to use Λ^* and Γ^* to shrink the monetary base. Without such an intervention, if UIP hold, the capital inflow will make the exchange rate appreciate over time (footnote 8 and section 7).

Faster expected devaluation will presumably reduce the desired share of domestic bonds in the foreign portfolio η^*. From (13), the domestic interest rate will have to rise (the domestic schedule moves to the right) to restore market balance. In (13*), if foreign wealth-holders switch even partially from domestic bonds into foreign money, the $(\phi^* + \eta^*)$ term will become negative, shifting the foreign schedule upward. In the new equilibrium, i will rise and there will be an ambiguous (probably small) shift in i^*.

In both (13) and (13*), dimensionally alert asset-holders will deflate expected depreciation by the current exchange rate e to create a rate of return ε/e comparable to the others in the model. For a given ε, a discrete (jump) depreciation of the spot rate e will increase η^*, forcing i to decline and i^* to shift either way. These responses could be reversed by depreciation effect on various terms in the market balance and net foreign asset equations, but in keeping with most of the literature, such wealth effects can be ignored.

Finally, notwithstanding then stocks of bonds and the real side of the economy with its transactions demands, in an extreme case the only relevant arguments in the domestic asset equation (13) could be i, i^*, e, and ε. Although uncovered interest rate parity applies to expectations about future values of the spot rate, the literature often postulates that contemporaneous partial derivatives force these four variables to be related in a UIP form such as $- i + (i^* + \varepsilon/e) = 0$. Because foreigners know as much about arbitrage as do domestic residents, this formula would be their asset market equation too. The whole world would have just one asset relationship. Non-market or institutional forces would have to set most asset prices and rates of return. This situation is far from the original goal of the portfolio balance model to determine all financial variables by contemporary market clearing only.

5. The Mundell–Fleming Model

Compared to the Portfolio Balance Model, the Mundell–Fleming model is an accounting mare's nest. It puts a flow goods market balance (the IS curve) together with a stock asset market equilibrium (the LM curve), and throws in part of (2) above as a BP relationship. Will this last equation be satisfied when goods and asset markets are in balance?

The answer is positive here, irrespective of the central bank interventions. The BP equation is not independent, i.e. there cannot be an external imbalance for an exchange rate adjustment to remove.

To demonstrate this result, the key assumption is that asset market balances are satisfied continuously over time, i.e. the existence of stock equilibria implies that flow equilibria exist as well. The relevant specification is in terms of flow-of-funds relationships from Table 1 which when supplemented by capital gains and losses are time-derivatives of balance sheets in Table 2[9]. The equilibrium condition needed from the real side is savings-investment balance. Then, the adjustment mechanisms to generate the relevant equality in IS equation (16) can be assumed to exist.

To be consistent with the presence of investment in cells (A, 5) and $(A^*, 5^*)$ of the accounting matrix in Table 1, private sectors must now be allowed to hold capital stocks. In the domestic economy the three asset demand functions (5)–(7) continue to apply, along with a stock demand for capital

$$\kappa\Omega - qPK = 0,$$

under $\mu + \eta + \phi + \kappa = 1$. The valuation ratio q adjusts to clear this equation, and may also enter as an argument in the investment demand function. This simple treatment of capital finance could be considerably expanded as in Franke and Semmler (1999), but it suffices in terms of preoccupations here with the exchange rate and balance of payments.

To demonstrate how Mundell–Fleming flow equations relate to portfolio balance, the relationship between the flows in Table 1 and the definitions of the change in domestic net foreign assets in (2) is shown here. It is consistent with any equilibrium theory of asset accumulation and portfolio selection — Keynesian, intertemporal optimization, etc.

The first point to recall is that from (2), foreign savings S_f is equal to the current account deficit,

$$S_f = [eP^*aX + i(T_f + R)] - [Pa^*X^* + ei^*(T_h^* + R), \tag{14}$$

and the flows of funds of the rest of the world with the domestic economy become

$$S_f + [e(\dot{T}_h^* + \dot{R}^*) - (\dot{T}_f + \dot{R})] = 0. \tag{15}$$

Foreign savings and capital inflow to domestic private sector and central bank are the foreign country's sources of funds. Flow acquisitions of domestic securities by its private sector and banks

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9) To keep the analysis simple, all variables are assumed to be continuously differentiable functions of time. In practice, both stock and flow variables can change discontinuously. But if in so doing they obey all relevant balance sheet and income statements, the transition from portfolio balance to Mundell–Fleming accounting goes through. The extensive theory and notation required to deal with such eventualities as well as changes in prices is best avoided here. Foley (1975) and Burgstaller (1994) take up the complications. In theory, accounting consistency between stock and flow variables is a means for linking the future to the present; in practice (in discrete time), it will be observed in sectoral balance sheet and flow of funds accounts because they are constructed that way.

are the uses of funds.

In other words, S_f equals the increase in domestic foreign debt $(\dot{T}_f + \dot{R})$ less the increase in its foreign holdings $e(\dot{T}_h^* + \dot{R}^*)$. For this Mundell–Fleming BP relationship to be independent, it must be possible for the domestic economy's increase in net foreign assets (the bracketed term in (15)) to differ from its current account surplus $-S_f$ when domestic and foreign IS and LM relationships are satisfied. Only in such circumstances will the exchange rate and some other variable have to play to restore the balance of payments for temporary equilibrium.

It is easy to see that this situation normally does not occur. First note that the sum of the flow of funds in rows (E)–(G) in Table 1 and (15) gives

$$S_h + S_g + S_f - PI = 0, \tag{16}$$

and the saving-investment balance applies. Because equality in (16) is assumed to be assured by real side IS equilibrium, only three of the four flows of funds (including (15)) can be independent relationships. They are further constrained by flow clearing of asset markets. The relevant equations appear in columns (8)–(10) and (9^*)–(10^*) in the accounting matrix, together with associated exchange conversions.

All asset markets can be assumed to clear in flow terms if the interest rates i and i^* are free to adjust, the banks satisfy their flows of funds restrictions, and the results of last section apply. Then the proof that the balance of payments must clear is trivial. Substitutions through the flows of funds under the assumption that goods market is in equilibrium (i.e. (16) is valid) immediately make sense of (15).

There is no need for the exchange rate or anything else to vary to ensure this equality will hold. In the standard textbook diagram, BP always passes through the IS/LM intersection, regardless of the value of e. As in the portfolio balance model, the triple intersection will occur whether or not central banks use flow transactions such as $\dot{\Gamma}^*$ and $\dot{\Lambda}$ (which could be incorporated in the accounting in straightforward way) to regulate changes in their holdings of domestic and foreign bonds.

It may be helpful to explore the implications of this result in more intuitive terms. If equality did not hold in (15), then in the double-entry bookkeeping of the flows of funds and flow asset balances some other equality would have to be violated. For example, if a country is running up external arrears by not meeting payment obligations on external debt outstanding so that the current account surplus $-S_f$ falls short of the bracketed term in (15).

There are two possible forms of repercussion on domestic flow asset market balances and flows of funds. One is that some other flow of funds relationship is not satisfied. The other is that some other flow of funds relationship is not satisfied.

Consider the other case. The obvious counterpart to a non-clearing balance of payments is the domestic bond market in columns (9) and (10^*) and rows (L) and (L^*) of the accounting matrix. The running up of external arrears would be reflected into a flow excess supply of domestic bonds — foreigners do not buy enough domestic securities to provide the country with money to meet its external obligations. Under such circumstances, a spot depreciation of appropriate degree could be

expected to erase the excess supply through a substitution effect and remove the disequilibrium.

The rub is that if other domestic financial markets are in equilibrium then this kind of adjustment is unnecessary, because the portfolio balance model shows that if the domestic money market clears then so will the bonds market. And with both money and bond markets in balance, there is no room in the accounting for an open balance of payments gap.

The other possibility is that the non-clearing balance of payments is reflected into another flow of funds relationship subject to the equilibrium condition (16). For example, as observed in recent Asian, Latin American, and Russian crises, situations in which the country is running up external arrears when the domestic private sector is undertaking investment projects that aren't working out (the sum of the terms in row (E) exceeds zero). The exchange rate realignment might reverse such simultaneous build-ups of external and internal bad debt. But at the macroeconomic level such situations are unusual. A banking sector would not provide non-performing loans to corporations usually. In harmonious times, the balance of payments evolves automatically from output and asset market equilibria.

Finally, taking exchange rate movements into account, net foreign assets is determined according to the relationship

$$\dot{N} = e(\dot{T}_h^* + \dot{R}^*) - (\dot{T}_f + \dot{R}) + \dot{e}(T_h^* + R^*) \tag{17}$$

If $\dot{N}$ were pre-determined, then (17) could be treated as an extra equation to be solved for $\dot{e}$ in a floating rate case in which central banks use flow interventions like $\dot{\Gamma}^*$ and $\dot{\Lambda}$ to control $\dot{R}^*$ and $\dot{R}$. But in line with the discussion of (1) above, there are no obvious economic forces that would make $\dot{N}$ anything but the passive sum of the variables on the right-hand side. Even if $\dot{N}$ were pre-determined, the faster capital inflows $\dot{T}_f$ in (17) might speed up exchange rate depreciation $\dot{e}$ in the counter-intuitive way. The usual Mundell–Fleming floating rate framework is not automatically drawn from consistent stock-flow accounting.

6. Comparative Statics of IS/LM

The model thus reduces to linked IS/LM framework for the two countries. Financial markets are described by equations (8a) and (13*), which clear (the central bank interventions and other shocks) asset market through adjustments in i and i*. Goods markets are described by (16) and its analog in the foreign country. Saving and investment functions can be assumed to respond to the variables: interest rates, levels of economic activity, profit rates emerging from the technology and institutions underlying V and V*, perhaps q and q*, and so on.

For present purposes, it makes sense to see how a Keynesian version behaves, with activity levels in both countries determined by effective demand. Because BP equations are not independent, the spot exchange rate has to be taken as a pre-determined variable in the short run. Here, according to usual literature, the home country is assumed to be small in goods markets, in that the foreign country's reserves stay constant, and in the sense of Figure 2.

Table 3 Signs of market responses in a linked IS/LM model

	X	i	X^*	i^*	e	ε
IS	–	–	+	0	±	0
LM	+	–	+	+	–	+
IS^*	0	0	–	–	±	0
LM^*	0	0	+	–	+	–

Table 3 gives signs of responses of excess demand of goods and bond excess supply functions to the endogenous variables such as output levels X and X^* and interest rates i and i^* in the two-country model as well as to e and ε. The IS row shows that as usual domestic excess demand is reduced by increases in both X and i. Higher foreign output X^* stimulates domestic demand via exports while changes in the foreign interest rate i^* are assumed to have no direct effects. In line with the possibility of contractionary depreciation (Krugman and Taylor, 1978), a higher value of e may either reduce or increase domestic aggregate demand. The faster expected depreciation ε has no direct effect (Table 3).

In the LM row, by increasing transactions demand for money, a higher X raises an excess supply of domestic bonds, while a higher i cut it back. Increases in X^* and i^* also raise excess supply, while e and ε have the effects discussed in connection with Figure 2. Foreign IS^* and LM^* schedules are not affected by output and interest rate changes in a small country.

Given the assumptions of Table 3, it is easy to work out how X and i respond to the other variables. (For simplicity, the only direct effects of e and in the IS and LM rows, without solving though IS^* and LM^* are considered.) Increases in both i^* and ε reduce X and raise i. Even if domestic and foreign bonds are close substitutes in domestic portfolios as in Figure 2, it will be true that $\partial i/\partial i^* < 1$ because the fall in X reduces excess supply of bonds. A higher X^* puts pressure on domestic financial markets and bids up i. The effect on X is ambiguous: export increases while domestic demand contracts due to a higher interest rate.

If depreciation is expansionary, a higher e raises X but has an ambiguous impact on i. The interest rate will rise if the output increase is strong and the wealth and substitution effects of depreciation on domestic asset equilibrium are weak. A weaker currency is supposed to take pressure off interest rates, but that outcome does not have to happen.

If depreciation is contractionary, it unambiguously reduces i. The feedback effect on effective demand leaves the final response of X unclear. Exchange rate appreciation in this case will be associated with a higher interest rate and possibly a reduction in output. Strong exchange rates, high interest rates, and slow growth have been characteristic of many emerging economies in the 1990s. If the fall in e is the driving force (because exchange rate appreciation is pursued as an anti-inflationary tool), the output and interest rate responses may reflect a situation in which effective demand is reduced or not strongly stimulated by depreciation.

So what happens to the balance of payments while these changes are going on? As in the portfolio balance model, reserve changes (created by central banks) will be the accommodating variables in both countries when their current accounts (driven largely by IS adjustments and interest rate

changes) and holdings of external bonds (driven by portfolio selections) shift over time. In the medium run, the current account will be the autonomous component of the balance of payments, so long as asset markets clear smoothly. Unless foreign portfolio preferences shift away from domestic bonds as its current account deficits or debt-service obligations become too large, there is nothing in the model to prevent a country from running an external deficit indefinitely. Perhaps the U.S. since the 1980s is a typical case.

7. Exchange Rate Dynamics and UIP

The exchange rate is not determined by temporary macro equilibrium conditions. It must evolve over time subject to the expectations about its future values. In a world of shifting and mostly unstable expectations, no simple dynamic theory is likely to emerge. The best candidate is UIP, but it does not reliably fit the data.

However, because UIP relies on arbitrage arguments which "should be true" and has interesting dynamic properties, a scenario based upon it is worth sketching as an illustration of the kinds of results that a standard hypothesis can generate in conjunction with a demand-driven IS/LM-based growth model operating along Keynesian lines. Of course, other model "closures" or causal structures are possible[10].

Under myopic perfect foresight about changes in the exchange rate with $\varepsilon = \dot{e} = de/dt$, the UIP formula (3) or (4) can be restated as a differential equation

$$\dot{e} = e\,(i - i^*) \tag{18}$$

In the two-country IS/LM framework, the interest rates i and i^* are functions of the exchange rate and other variables. A long-run equilibrium rate $\bar{e}$, which may or may not be easily attained, is defined by the condition $i = i^*$. Together with the IS/LM model, (18) makes up a well-defined dynamic system. On the right-hand side, i and i^* are functions of e, $\dot{e}$ and other variables Q, so (18) becomes $\dot{e} = f(e, \dot{e}, Q)$. This equation can be converted to the standard form $\dot{e} = g(e, Q)$. Assuming myopic perfect foresight, if the portfolio balance model really did have three independent equations among i, i^*, e, $\dot{e}$, and other variables, it would itself be a dynamic system for the exchange rate, inconsistent in general with UIP, not to mention full intertemporal arbitrage and

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10) For example, both the celebrated model by Dornbusch (1976) and the one here stand on IS and LM (but not BP) foundations. But for Dornbusch, output is set by Say's Law (transiently relaxed by "sticky price" short-term output dynamics); the money market balance is combined with UIP and regressive exchange rate expectations around Purchasing Power Parity (an "extra equation") to make e depend on the real money supply M/P; changes in P over time are driven by the difference between aggregate demand and output; and since this difference is non-zero away from the steady state the short-run current account is endogenous and implicitly offset by capital flows or reserve movements which are sterilized to let the central bank control M. Because it obeys PPP and the quantity theory in the long run, the model is more closely tied to "fundamentals" than the one presented here.

optimization[11].

Differential equations incorporating myopic perfect foresight in growth and financial models are usually unstable. An asset price feeds back positively into its own rate of growth, giving rise to saddle-path dynamics (under appropriate transversality conditions) or a bubble.

However, at an initial equilibrium at which $i = i^*$ and $e = \bar{e}$, $d\dot{e}/de < 0$ in (18) if devaluation reduces the domestic interest rate as discussed above. This local stability opens up possibilities for cyclical exchange rate dynamics not present in most perfect foresight.

Before proceeding to it, however, it is worth noting how UIP supports Mundell–Fleming conclusions. From the comparative statics discussed above, a capital inflow with the money supply held constant is likely to make i fall and i^* rise. If the differential equation (18) is stable, the exchange rate will decline over time, driving the interest rates back together.

The "floating rate" scenario of a capital inflow leading to appreciation applies dynamically, except that e is not floating in the traditional meaning, but is being determined in forward markets through UIP.

As an illustration of a full dynamic system, how the exchange rate interacts with domestic government debt can be traced. The relevant state variable from the IS/LM system is D = T/PK, in which T is the outstanding stock of government bonds, P is the domestic price level, and K is the domestic capital stock. Ignoring depreciation, K increases over time according to the rule $\dot{K} = I = gK$ where I is the level of investment made by business and g is the growth rate of K. Indeed, g can be treated as the model's investment function. As noted above, it can be assumed to depend on the interest rate i, the output/capital ratio u = X/K as a measure of "capacity utilization," and the profit rate $r = \pi u$, where π is the share of profits in total income. Alternatively, the valuation ratio q can be taken as the principal argument of the function g. The time derivative of T is $\dot{T} = \gamma PK + iT$ where γ is the government's "primary deficit" (outlays apart from interest payments less revenue) as scaled to the value of the capital stock. Using the foregoing information, a differential equation for D becomes

$$\dot{D} = \gamma \dot{D} + [(i - \hat{p}) - g])D, \tag{19}$$

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11) As observed by Branson and Handerson (1985), there were many papers in the 1970s and early 1980s devoted to dynamic analysis of a portfolio balance model augmented by IS and BP relationships. A typical "finding" was that the dynamic system could be unstable if domestic net foreign assets were negative. But this argument was incorrect because it assumed the spot rate could be set by portfolio balances and also treated the BP equation as being an independent restriction on the dynamic system.

12) The Dornbusch model uses "rational" regressive expectations on e and a fixed i^* to determine i through UIP and introduces unstable dynamics in the price inflation equation. For future reference, a possible instability in (18) due to expectational effects in asset demands is worth noting. With myopic perfect foresight, $\dot{e}$ equals expected depreciation ε and (as noted in the text), shows up as a determinant of the interest rates on the right-hand side of (18). Differentiation gives $d\dot{e}/de = (i - i^*) + e[\partial i/\partial e + (\partial i/\partial \dot{e})(d\dot{e}/de)]$ where the $\partial i/\partial e$ term comes from the IS/LM system. Minor change shows that $d\dot{e}/de = [1 - e(\partial i/\partial \dot{e})]^{-1} e(\partial i/\partial e)$. When $0 < e(\partial i/\partial \dot{e}) < 1$ (the traditional inelastic expectations logic), the UIP differential equation is locally stable. But strong expectational effects could make $d\dot{e}/de > 0$ even when di/de is negative.
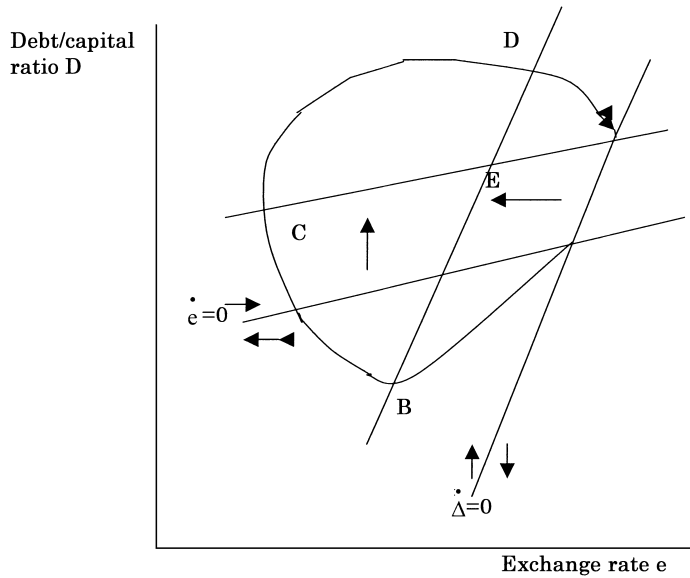
Debt/capital
ratio D

Fig. 3　Adjustment dynamics of the exchange rate and debt/capital ratio (after "a permanent" monetary expansion)

where $\hat{P} = \dot{P}/P$ is the inflation rate. The steady state value of the debt/capital ratio is $\overline{D} = \gamma/[g - (i - \hat{p})]$. As is well-known, an economy with $\gamma > 0$ can only sustain a stable debt/capital ratio when its capital stock growth rate exceeds the real interest rate. This condition has tended to fail recently in industrialized economies (and fail strongly in developing debtor countries such as Argentina). We will assume, however, that at least some of the time g can exceed $(i - \hat{P})$.

Another complication is that the components of the bracketed term on the right-hand side of (19) depend on D. A higher value is analogous to an increase in T in (8). The excess supply of bonds increases, forcing their interest rate i to rise. In most IS/LM models with an inflation equation adjoined, the inflation rate $\hat{p}$ and growth rate g would go down. The derivative of the bracketed term in (19) with respect to D would be positive, which could make the total derivative $d\dot{D}/dD$ positive around a steady state with $\overline{D} > 0$. This potentially unstable case is the focus of the following discussion.

Effects of the exchange rate e on $\dot{D}$ in (19) go through the interest rate directly, as well as through shifts in g and $\hat{p}$ induced by interest and output changes. If $\partial i/\partial e < 0$ in the IS/LM framework, a direct negative impact of e on $\dot{D}$ is derived. If the inflation and growth rates rise with a lower i, it is clear that $d\dot{D}/de < 0$. Finally, since the more D push up i, $d\dot{e}/dD > 0$ in (18). Figure 3 is a phase diagram for the system (18)–(19) in which there is a chance for cyclical stability. The initial equilibrium at which $\dot{D} = \dot{e} = 0$ resides at point A. Suppose that there is a permanent monetary expansion, pushing down i. To bid up the interest rate again, D would have to rise and e to fall. The curves shift as illustrated in Figure 3.

Along the dynamic trajectory, the lower interest rate initially sets off exchange appreciation and a declining debt/capital ratio. The falling exchange rate begins to push up the interest rate, increasing

the bracketed term in (19) until D starts to rise at point B. Both variables are now putting upward pressure on the interest rate. When i rises above i^* in (18), the exchange rate begins to depreciate. The trajectory may or may not converge to the new equilibrium at E[13]. Even if it does, the economy is likely to go through cycles. An initial monetary contraction would create depreciation-then-appreciation exchange rate history. Since the late 1970s, the U.S. external deficit seems to have been accompanied by such cycles over 10 years period.

8. Conclusions

The results from the above argument illustrates that a properly specified open macro economic model contains interesting possibilities. Further, it would carry over to intertemporal models which incorporate UIP, but replace effective demand with Say's Law and derive a private savings (= investment) rate from a Ramsey-style dynamic optimization. Even with these changes, intertemporal models have to satisfy accounting relationships like those in Table 1 and Table 2. Their dynamics are less complicated than the trajectory of Figure 3, and as shown in footnote 12, outright instabilities are also possible. There is no reason to expect monotypic or saddle-point stability. This observation much differs from early predictions based on the portfolio balance model and Mundell–Fleming model. In one familiar example based on the former, suppose that the country runs a current account deficit. Its reserves will fall, leading to monetary contraction and a higher interest rate. If UIP applies, the exchange rate will depreciate over time, presumably leading to a better trade performance and a new long-run equilibrium in which the current account is balanced and exchange and interest rates are stable. In the traditional model, stock-flow adjustments with capital mobility will generally move the exchange rate in the right direction to eliminate a current account deficit in the long run (Blecker, 1999).

This story resembles the history of David Hume's price-species-flow-mechanism in which a current account deficit stimulates price adjustments such as deflation that will make the deficit disappear in the Gold Standard System. But in fact, there was little evidence and reason for such adjustments to happen. In (18), a floating exchange rate has no fundamentals such as a real rate of return or a trade deficit that can make it self-stabilizing. This observation helps explain why all "fundamentals-based" exchange rate models have been failed empirically. The evidence is well covered by Frankel and Rose (1995).

Although most of the emphasis here has been focused on Keynesian specifications, points raised could be applied to the monetarist econometric models as well. They usually start out correctly by postulating just two equations linking money demand to the interest rate and price level in domestic and foreign countries in which exchange rate and its expected change are not included as arguments in these functions, though they should be.

The models add the assumption that money markets clear through price adjustments with supplies predetermined, i.e. price levels are set by the equation of exchange rate modified for interest

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13) That is, a critical point E of the system (18)–(19) in Figure 3 can be either a stable or unstable focus.

rate effects. Purchasing Power Parity is then used as an "extra equation" to determine the exchange rate from the two country's price levels. Because a tight link between an economy's money supply, its price level and PPP is not strongly supported by the data, it is not surprising that these formulations seem to have failed. Similar observations apply to extensions based on UIP, rational expectations, gradual price adjustment, and so on.

The conclusion is that e can only float against its own expected future values and the interest rates. In the real world, such expectations are determined chiefly by intrinsically unpredictable factors and non-rational behaviors of investors. As shown roughly in this technical paper, open macroeconomic system that has fewer temporary equilibrium conditions has been considered to widen a range of possibilities of exchange rate determination. This possibility could be explored further.

At the close of this paper, the limit of this paper should also be stressed. This paper is mostly technical one which focuses on the inconsistency of exact balance sheet and Portfolio Balance, Mundell–Fleming, Uncovered Interest Rate Parity models. From accounting point of view, the number of equations falls short of the number of variables from orthodox general equilibrium point of view. This drawback seems chiefly come from the lack of variable which overtly stand for the formation process of expectation of investors.

As is well known, the temporary fare value of the exchange rate is supposed to subject to rather strict condition. i.e. the stochastic distribution of the future value will be subject to log-normal. If this condition which is drawn by perfect foresight hypothesis and rational behavior were not hold, the most of the logic would likely to fail. From this reason, the future research could more emphasize the elucidation of the structure and formation process of expectation, both rational and non-rational. If the structure of expectation would be incorporated, it will enable to explain a part of erratic gross capital movement and also fill a part of the missing link between cross-border balance sheet and open macro economic models. Unless this problem clarified, any open macroeconomic theories about the exchange rate would not likely to outperform the naïve random walk theory.

References

1) Blecker, Robert A. (1999) "Taming Global Finance: A Better Architecture for Growth and Equity," Washington DC: Economic Policy Institute.
2) Blecker, Robert A. (2002) "Capital Mobility, Macroeconomic Imbalances, and the Risk of Global Contraction," in John Eatwell and Lance Taylor (eds.) "International Capital Markets: Systems in Transition.", New York: Oxford University Press.
3) Branson, William H., and Dale W. Henderson (1990) "The Specification and Influence of Asset Markets." in Ronald W. Jones and Peter B. Kenen (eds.) "Handbook of International Economics. Vol. 2." Amsterdam, North-Holland.
4) Burgstaller, Andre (1994) "Property and Prices: Towards a Unified Theory of Value", New York, Cambridge University Press.
5) Dornbusch, Rudiger (1976) "Expectations and Exchange Rate Dynamics." Journal of Political Economy, 84.
6) Eatwell, John, and Lance Taylor (2000) "Global Finance at Risk: The Case for International Regulation." New York, New Press.

7) Fleming, J. Marcus (1962) "Domestic Financial Policies under Fixed and Floating Exchange Rates," IMF Staff Papers, 9.

8) Foley, Duncan K. (1975) "On Two Specifications of Asset Equilibrium in Macroecomomic Models." Journal of Political Economy, 83.

9) Foley, Duncan K., and Miguel Sidrauski (1971) "Monetary and Fiscal Policy in a Growing Economy." New York, Macmillan.

10) Franke, Reiner, and Willi Semmler (1999) "Bond Rate, Loan Rate, and Tobin's q in a Temporary Equilibrium Model of the Financial Sector." Metroeconomica, 50.

11) Frankel, Jeffrey A., and Andrew K. Rose (1995) "Empirical Research on Nominal Exchange Rates." in Gene Grossman and Kenneth Rogoff (eds.), Handbook of International Economics, Vol. 3, Amsterdam: North-Holland.

12) Godley, Wynne (1996) "A Simple Model of the Whole World with Free Trade, Free Capital Movements, and Floating Exchange Rates," The Jerome Levy Institute, Bard College.

13) Isard, Peter (1995) "Exchange Rate Economics." Cambridge: Cambridge University Press.

14) Kareken, John, and Neil Wallace (1981) "On the Indeterminacy of Equilibrium Exchange Rates," Quarterly Journal of Economics.

15) Kenneth Rogoff (2002) "Dornbush's Overshooting Model After Twenty-Five Years" IMF Working Papers, 39, IMF.

16) Keynes, John Maynard (1923) "A Tract on Monetary Reform." London, Macmillan

17) Keynes, John Maynard (1936) "The General Theory of Employment, Interest, and Money." London, Macmillan.

18) Krugman, Paul, and Taylor, Lance (1978) "Contractionary Effects of Devaluation." Journal of International Economics vol. 8.

19) Mundell, Robert A. (1963) "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," Canadian Journal of Economics and Political Science, 29.

20) Obstfeld, Maurice, and Kenneth Rogoff (1995) "The Intertemporal Approach to the Current Account." in Gene Grossman and Kenneth Rogoff (eds.), Handbook of international Economics, Vol. 3, Amsterdam, North-Holland.

21) Ramsey, Frank P. (1928) "A Mathematical Theory of Saving," Economic Journal, 38.

22) Rosensweig, Jeffrey A., and Lance Taylor (1990) "Devaluation, Capital Flows, and Crowding-Out: A CGE Model with Portfolio Choice for Thailand." in Lance Taylor (ed.) Socially Relevant Policy Analysis, Cambridge MA: MIT Press.

23) Salter, W. E. G. (1959) "Internal and External Balances: The role of Price and Expenditure Effect." Economic Record, 35.

24) Swan, Trevor (1960) "Economic Control in a Dependent Economy." Economic Record, 36.

25) Tobin, James (1969) "A General Equilibrium Approach to Monetary Theory." Journal of Money, Credit, and Banking.

26) Williamson, John, and Chris Milner (1991) "The World Economy," New York: New York University Press.